Commutators on generalized power series spaces over non-Archimedean fields

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The generalized power series spaces $D_f(a,r)$ over non-Archimedean fields are the most known and important examples of non-Archimedean nuclear Fréchet spaces. The *commutator* of a pair of operators A and B on a locally convex space E is given by $[A,B] := AB - BA$. An operator T on E is said to be a commutator if T can be expressed in the form $T = [A,B]$ for some operators A and B on E.

We prove among other things the following:

(I) Every operator on $D_f(a,r)$ is a commutator, if (1) $r \in \{0,\infty\}$ and $\sup_n[a_{2n}/a_n] < \infty$ or (2) $r \in (-\infty,0) \cup (0,\infty)$, $\lim_n[a_{2n}/a_n] = 1$ and f is rapidly increasing.

(II) If $\lim_n[a_{n+1}/a_n] = \infty$ and an operator T on $D_f(a,r)$ is a commutator, then T is bounded. In particular, the identity operator on $D_f(a,r)$ is not a commutator, if $\lim_n[a_{n+1}/a_n] = \infty$.